Roll No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] **Total No. of Pages :02**

Total No. of Questions : 07

B.Sc. (CS) (Sem.–6)

LINEAR ALGEBRA

Subject Code :BCS-602

M.Code :72782

Date of Examination : 06-07-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1. Answer the following in short :

a) Find the value of K so that the set of vectors (K, 1,1), (0,1,1), (K, 0, K) is linearly dependent?

b) Define basis and dimension of vector space.

c) Let $W_1$and W_2 be two subspaces of $R^3(R)$ generated by

$$\left\{(1,3-2,2,3),(1,4,-3,4,2),\left(1,\frac{3}{2},\frac{-1}{2},-1,\frac{9}{2}\right)\right\} \text{ and}$$

$$\left\{(1,3,0,2,1),(1,5,-6,6,3),\left(1,\frac{5}{2},\frac{3}{2},-1,\frac{1}{2}\right)\right\}$$ respectively. Find dimensions of W_1+W_2.

d) Define Ring.

e) Define vector subspaces.

f) State existence theorem for basis.

g) Define Nullity.

h) Define linear transformation.

i) Define isomorphism

j) Define quotient space.

SECTION-B

2. a) If S is a non empty subset of a vector space V, then $[S]$ is the smallest subspace of V containing S.

   b) In the complex vector space V_2^c show that

   $(1 + i, 1 - i$ belongs to $(1 + i, 1), (1,1 - i)$

3. Let T be a linear operator defined by $T(x, y) = (4x - 2y, 2x + y)$. Find the matrix of T relative to the basis $B = \{(1,1); (-1,0)\}$

4. Show that the transformation $T:R^3 \to R^3$ defined by $T(x, y, z) = (x + y, y + z, z + x)$ is linear.

5. State and prove Rank Nullity theorem.

6. Let U and V be two subspaces of vector space V and $Z = U + W$, then $Z = U \oplus W$ iff the following condition is satisfied:

   Any vector $z \in Z$ can be expressed uniquely as the sum, $z = u + w, u \in U, w \in W$.

7. a) Find the largest linearly independent subset whose span is $[S_4]$.

   b) Prove that in an n-dimensional vector space V, any set of n linearly independent vectors is a basis.

NOTE : Disclosure of identity by writing mobile number or making passing request on any page of Answer sheet will lead to UMC case against the Student.