

Roll No.

Total No. of Pages :03

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B.Tech.(Agriculture Engg./Automation & Robotics/Automation Engg./Civil Engg./Computer Science & Engg./Electrical & Electronics Engg./Electrical Engg./Electronics & Communication Engg./Electronics & Electrical Engg./Information Technology/Mechanical Engg.)/
B.Tech. (CSE/ECE) (PIT)(Sem.-1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Date of Examination : 01-07-22

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

Solve the following :

- 1) What is maximum value of function $f(x) = -x^2$.
- 2) Find the equation of normal line to the surface $xyz = 6$ at $(1, 2, 3)$.
- 3) Show that the function $f(x, y) = \begin{cases} \frac{xy}{2y^2 + x^2}; & (x, y) \neq (0, 0) \\ 0; & (x, y) = (0, 0) \end{cases}$ is discontinuous at $(0, 0)$.
- 4) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\log\left(x - \frac{\pi}{2}\right)}{\tan x}$.
- 5) Calculate approximate value of $\sqrt{10}$ to two decimal places by Taylor's theorem.

- 6) Evaluate $\int_0^1 \int_1^a \frac{1}{xy} dy dx$.
- 7) Examine the nature of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \infty$.
- 8) Define orthogonal matrices with example.
- 9) Show that $(1, 1, 2)$ is an eigen vector of the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ corresponding to the eigen value 2.
- 10) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

SECTION-B

- 11) a) Verify Cauchy's mean value theorem for $f(x) = \log x, g(x) = \frac{1}{x}$ in $[1, e]$.
- b) Apply Maclaurin's theorem with Lagrange's remainder to function $f(x) = \cos x$.
- 12) Discuss the convergence of the following improper integral
- a) $\int_0^{\infty} \frac{1}{b^2 x^2 + a^2} dx$ b) $\int_1^2 \frac{x+1}{\sqrt{x-1}} dx$.
- 13) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.
- 14) a) Evaluate by changing the order of integration of $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 - y^2}} dy dx$.
- b) Find the volume enclosed between the cylinders $x^2 + y^2 = 2ax$ and $z^2 = 2ax$.

SECTION-C

15) a) Discuss the convergence or divergence of the series $\sum \frac{n^p}{(n+1)^q}$.

b) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{[\log(\log n)]^n}$.

16) a) Test the convergence of $1 + \frac{(1+\alpha)}{(1+\beta)} + \frac{(1+\alpha)(1+2\alpha)}{(1+\beta)(1+2\beta)} + \frac{(1+\alpha)(1+2\alpha)(1+3\alpha)}{(1+\beta)(1+2\beta)(1+3\beta)} + \dots$

b) Discuss the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{n+\sqrt{n}}{n^2-n}$.

17) a) Use Gauss Jordan method to find the inverse of a matrix $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$.

b) Find a matrix B which transforms $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into a diagonal form.

18) Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$ has

- a) a unique solution,
- b) no solution,
- c) infinitely many solutions.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.