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Total No. of Questions : 09

B.Tech.(Food Technology)(Sem.–1)

MATHEMATICS-I

Subject Code :BTAM-106-18

M.Code :75368

Date of Examination : 01-07-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B &C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B& C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION-B &C.**

SECTION-A

1. Answer briefly :

a) Find characteristic equation of A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$.

b) Find the gradient of the function $\phi = y^2 + xz$ at (–1,2, –2).

c) Find div $\vec{F}$, $if\ \vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.

d) Evaluate $\begin{vmatrix} 2 & -3 & 4 \\ 5 & 1 & -6 \\ -7 & 8 & -9 \end{vmatrix}$.

e) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$, Find AB.

f) State Green's theorem.

g) Prove that product of two orthogonal matrices is orthogonal matrix.

h) Define irrotational Vectors. Give example.

i) Find curl $\vec{r}$, if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

j) Define skew symmetric matrix. Give example.

SECTION-B

2. a) Find Eigen value & Eigen vector of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

b) Write matrix A given below as the sum of a symmetric and a skew symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$$

3. a) Solve by Cramer's rule $5x - 7y + z = 11$, $6x - 8y - z = 15$, $3x + 2y - 6z = 7$

b) Determine ω, μ such that the following equations:

$2x - 5y + 2z = 8, 2x + 4y + 6z = 5, x + 2y + \omega z = \mu$.

(i) a unique solution, (ii) no solution, (iii) infinitely many solutions.

4. a) Solve by Gauss Jordan method, the system of equations

$x + 2y + z = 2$, $3x + y - 2z = 1$, $4x - 3y - z = 3$, $2x + 4y + 2z = 4$.

b) Are the vectors (2, –1,3,2), (1,3,4,2), (3, –5,2,2) linearly dependent? If yes, find relation between them.

5. a) Prove $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$ is an orthogonal matrix.

b) Diagonalize $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

SECTION-C

6. a) If $\vec{F} = 3xy\,\hat{i} - y^2\,\hat{j}$. Evaluate $\int_C \vec{F}.\vec{dr}$, where C is the curve in xy plane $y = 2x^2$ from (0,0) to (1,2).

b) Compute $\iint_S \vec{F}.\hat{N}\,dS$ where $\vec{F} = 18z\,\hat{i} - 12\,\hat{j} + 3y\,\hat{k}$ and S is the portion of the plane $2x + 3y + 6z = 12$ in the first octant.

7. a) Prove that $\nabla.(\vec{A} \times \vec{B}) = \vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$.

b) Find directional derivatives of $f = x^2 + y^2 + z^2$ at a point A (2, 2, 1) in the direction of $2\hat{i} + 2\hat{j} + \hat{k}$.

8. Verify Greens theorem for $\oint_C (x^2 - \cosh y)dx + (y + \sin x)dy$, where C is the boundary for a rectangle whose vertices are O (0,0), A(π, 0), B(π, 1), C(0,1).

9. a) If $\vec{r}$ has constant magnitude but variable direction, then prove that $\vec{r}.\frac{d\vec{r}}{dt} = 0$

b) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $2\hat{i} + 3\hat{j} + 6\hat{k}$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.