Roll No. [| | | | | | | | | | |] **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. Mathematics (2018 Batch) (Sem.-2)

ALGEBRA-II

Subject Code : MSM-201-18

M.Code : 75962

Date of Examination : 04-07-22

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. **Attempt the following :**

   a) State division algorithm in polynomial ring $R[x]$.

   b) Show that $x^3 - x - 1 \in Q[x]$ is irreducible over Q.

   c) Prove that every finite extension of field is algebraic extension.

   d) Define fixed field of group of automophisms with suitable example.

   e) Find the degree of $Q\left(\sqrt[3]{3}, \sqrt[4]{5}\right)$ over Q.

SECTION-B

2. Let R be a Unique Factorization Domain. Then show that polynomial ring R[x] over R is a Unique Factorization Domain.

3. a) Let $f(x) \in F[x]$ be a non constant polynomial. Show that there exists an extension E of F in which $f(x)$ has a root.

   b) Show that the product of two primitive polynomials is primitive.

4. Let E be an algebraic extension of F and $\sigma : E \rightarrow E$ be an embedding of E into itself over F. Then, show that σ is an automorphism. Also prove an element α of extension K is algebraic over F if and only if $[F(\alpha) : F]$ is finite.

SECTION-C

5. a) State and prove fundamental theorem of Algebra.

   b) Show that Galois group of $x^4 + 1 \in Q[x]$ is the Klein four-group.

6. Let E be a finite separable extension of a field F. Show that following are equivalent :

   a) E is normal Extension of F.

   b) F is a fixed field of G(E/F).

   c) [E : F] = | G(E/F)|

7. Show that there exists an algebraically closed field K containing F as a subfield. Also, prove that the splitting field of $x^3 + x^2 + 1 \in Z/(2)[x]$ is a finite field with 8 elements.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.