

Roll No.

Total No. of Pages : 02

Total No. of Questions : 07

M.Sc. Mathematics (Sem.-4)

DISCRETE MATHEMATICS

Subject Code : MSM-501-18

M.Code : 77871

Date of Examination : 04-07-22

Time : 3 Hrs.

Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions each.
3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. Write short notes on :

- a) Prove that the statement $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
- b) Consider the lattice D_{1001} , find the complement of each element of D_{1001} .
- c) Find the solution of the homogeneous recurrence relation
$$y_{k+4} + 4y_{k+3} + 8y_{k+2} + 8y_{k+1} + 4y_k = 0$$
- d) Define Chromatic number. What is the chromatic number of K_n ?
- e) Define group codes.

SECTION-B

2. a) Consider $e : B^3 \rightarrow B^7$ defined by

$e(000) = 0000000$; $e(001) = 0010110$; $e(010) = 0101000$; $e(011) = 0111110$;
 $e(100) = 1000101$; $e(101) = 1010011$; $e(110) = 1101101$; $e(111) = 1111011$.

Show that encoding function $e : B^3 \rightarrow B^7$ is a group code.

- b) Using the properties of Boolean algebras, prove that for each x in a algebra if $x + y = 1$ and $xy = 0$ then $y = x$.

3. a) Suppose the preorder and inorder traversals of a binary tree T yields the following sequences of nodes when the following is given

Preorder : G, B, Q, A, C, K, F, P, D, E, R, H

Inorder : Q, B, K, C, F, A, G, P, E, D, H, R

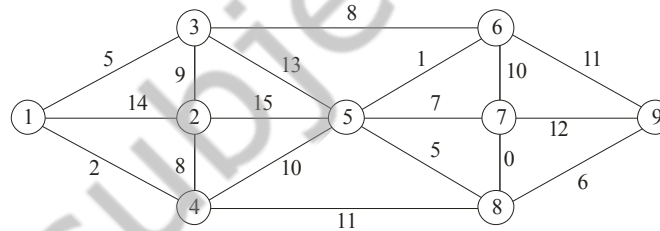
- i) Draw the diagram of T.
 ii) Find the depth d of T.
- b) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.
4. Let $D_{70} = (1, 2, 5, 7, 10, 14, 35, 70)$, the divisors of 70. Show that D_{70} is a Boolean algebra under the binary operations '+' and '.' and defined by

$$\left. \begin{aligned} a+b &= \text{l.c.m.}(a,b) \\ a.b &= \text{g.c.d.}(a,b) \end{aligned} \right\}$$

$a' = \frac{70}{a}$, where 1 is a zero element and 70 is a unit element.

SECTION-C

5. Find a minimum spanning tree of the weighted graph shown below :



6. a) Solve the recurrence relation $t_n = -3t_{n-1} - 3t_{n-2} - t_{n-3}$, $n \geq 3$, subject to $t_0 = 1$, $t_1 = -2$, $t_2 = -1$.
- b) Find the minimum distance of (2, 4) encoding function $e : B^2 \rightarrow B^4$ defined as $e(00) = 0000$, $e(10) = 0110$, $e(01) = 1011$, $e(11) = 1100$.
7. a) Prove that a planar and a connected graph has a vertex of degree less than or equal to 5.
- b) The chromatic number of graph c_n , where c_n is the cycle with n vertices is either 2 or 3.

NOTE : Disclosure of identity by writing mobile number or making passing request on any page of Answer sheet will lead to UMC case against the Student.