Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 07

M.Sc. (Mathematics) (Sem.-4)

ADVANCED COMPLEX ANALYSIS

Subject Code : MSM-505-18

M.Code : 77875

Date of Examination : 13-07-22

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION-B & C have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. Answer the following :

a) Define genus and order of entire function.

b) State Jensen's formula.

c) Define starlike functions.

d) State Montel's theorem.

e) Define subharmonic and superharmonic functions.

SECTION-B

2. State and prove Runge's theorem

3. a) Prove Walli's formula $\frac{\pi}{2} = \prod_{n=1}^{\infty} \frac{(2n)^2}{(2n-1)\,(2n+1)}$.

b) Show that if lim sup $|b_n| < \infty$ then $\sum_{n=1}^{\infty} \left(\frac{r}{a_n}\right)^{k_n} \frac{b_n}{a_n}$ converges absolutely for all $k_n = n$ for all n.

4. Let y : [0, 1] → ℂ be a path from a to b and let $\{(f_t, D_t) \mid 0 \leq t \leq 1\}$ be an analytic continuation along γ. There is a number $\in > 0$ such that if $\sigma : [0, 1] \to \mathbb{C}$ is any path from a to b with $|\gamma(t) - \sigma(t)| < \in$ for all $t \in [0, 1]$, and if $\{(g_t, B_t) \mid 0 \leq t \leq 1\}$ is any continuation along σ with $[g_0]_a = [f_0]_a$; then $[g_1]_a = [f_1]_a$.

SECTION-C

5. State and prove Hurwit'z theorem.

6. a) Solve the Dirchlet problem for the disc D = Δ(0, 2) with the boundary data $\phi(z) = x_{2\,+}\, 2xy^2$.

   b) If U < C is simply connected and $0 \notin$ U, then there exist a univalent map $f : U \to \mathbb{C}$ such that $f(z)^2 = z$.

7. State and Prove Riemann mapping theorem.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.