Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. (Mathematics) (Sem.-4)

THEORY OF LINEAR OPERATORS

Subject Code : MSM-509-18

M.Code : 77879

Date of Examinaton : 18-07-22

Time : 3 Hrs. Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. Write briefly :

(a) Define Spectrum and Resolvent.

(b) Explain representation of the resolvent $R_\lambda(T)$ by a power series.

(c) Define compact linear operator on normed spaces X and Y.

(d) Let X and Y be normed spaces. Prove that zero operator on any normed space is compact.

(e) Let H be a complex Hilbert space, T be a bounded self adjoint operator on H. Show that all the eigen values of T are real.

SECTION-B

2. State and prove spectral mapping theorem.

3. Prove that the resolvent set $\rho(T)$ of a bounded linear operator T on a Banach space X is open, hence its complement the spectrum $\sigma(T)$ is closed.

4. If H be the set of all invertible elements of a complex Banach algebra then show that the mapping H → H defined by $x' \to x^{-1}$ is continuous.

SECTION-C

5. Let $\{T_n\}$ be a sequence of compact linear operators from a normed linear space X into a Banach space Y. If $\{T_n\}$ is uniformly convergent, then show that the limit operator T is compact.

6. Let X be a normed space and T be a compact linear operator. Show that the set of eigen values of T is countable.

7. Let H be a complex Hilbert space and T be a bounded self adjoint operator on H. Then show that a number $\lambda \in \rho(T)$ (resolvent set of T) iff $\ni$ c > 0 such that $\|T_\lambda x\| \geq c\|x\|$ for every $x \in H$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.