Roll No.

Total No. of Pages: 03

Total No. of Questions: 09

B.Tech. (AI&ML / CE / CSE / IT / Internet of Things and Cyber Security including Block Chain Technology) (Sem. – 4)

DISCRETE MATHEMATICS

Subject Code: BTCS-401-18

M Code: 77626

Date of Examination : 04-01-2023

Time: 3 Hrs. Max. Marks: 60

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Answer the following:

a) Define Cartesian product (AXB) of two non empty sets. How many different elements does A × B have if A has m elements and B has n elements?

b) What are the contrapositive, the converse, and the inverse of the conditional statement. The home team wins whenever it is raining?

c) Prove that if n is an integer and n^2 is odd, then n is odd.

d) What is the power set of the set $A = \{0,1,2\}$?

e) How many people among 200000 are born at the same time. Use pigeonhole principle to find it.

f) State fundamental theorem of arithmetic.

g) Let G be a simple graph with n vertices.Show that G has at most $\frac{n(n-1)}{2}$ edges.

h) Define an abelian group.

i) Let a, b be elements in a Boolean Algebra, prove that $a + a^*b = a$

j) Define vertex colouring in a graph.

SECTION-B

2. Let p and q be the propositions

   p:You drive over 65 miles per hour. q:You get a speeding ticket

   Write these propositions using p and q and logical connectives (including negations).

   a) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

   b) You get a speeding ticket, but you do not drive over.

3. Among 100 Students, 32 study Mathematics, 20 study Physics, 45 study English, 15 study Mathematics and English, 7 study mathematics and Physics, 10 study Physics and English and 30 do not study any of the three subjects. Find the number of students studying all three subjects.Find also the number of students studying exactly one of the three subjects.

4. Let A be the set of integers and R be the relation defined on AxA by $(a, b)R(c, d)$ if ad=bc. Prove that R is an equivalence relation.

5. Define the following with suitable examples:

   a) Isomorphic graphs

   b) Eulerian graph

   c) Hamiltonian graph

   d) weighted tree

6. Show that each subgroup of an abelian group is normal subgroup.

SECTION-C

7. a) Show that these statements about the real number x are equivalent:

      a) x is irrational

      b) $3x + 2$ is irrational

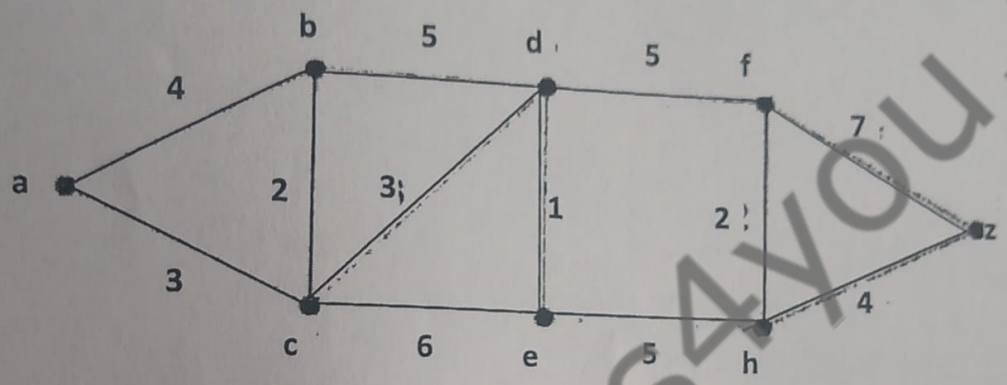
      c) $x/2$ is irrational

b) Find how many arrangements can be made with the letters of the word 'MATHEMATICS? In how many of them:

i) consonants occur together

ii) vowels do not occur together?

8. a) Find the length of the shortest path between a and z in the given weighted graph. Explain the algorithm used for finding this path.



b) Prove that a field is an integral domain. Is the converse true?

9. a) Write the function $x \vee y'$ in the disjunctional normal form in three variables x, y and z

b) Prove by Boolean algebra: $A + B.C = (A + B).(A + C)$

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.