Roll No. | | | | | | | | | | | | | **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. (CE/ CSE/EE/ECE/ME) (Sem–1)

ENGINEERING MATHEMATICS-I

Subject Code : BTAM-101

M.Code : 54091

Date of Examination : 07-06-2023

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Solve :

a) If $\vec{F} = (x+y+1)\,\hat{i} + \hat{j} - (x+y)\,\hat{k}$. Then show that $\vec{F}$. curl $\vec{F} = 0$.

b) Find $\frac{\partial y}{\partial x}$ if $z = \sin^{-1}\frac{y}{x}$.

c) State Gauss Divergence theorem

d) If u = x sin y and v = y sin x, find $\frac{\partial(u,v)}{\partial(x,y)}$.

e) If an error committed in measuring the side of square is 2 %. Find the error in calculating the area.

f) Evaluate $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y}\,dy\,dx$.

g) Find the gradient of the function $\phi = x^3 + y^3 + 3xyz$ at (1, –2, –1).

h) A fluid motion is given by $\vec{v} = (y\sin z - \sin x)\,\hat{i} + (x\sin z + 2yz)\,\hat{j} + (xy\cos z + y^2)\,\hat{k}$. Is the motion irrotational?

i) Obtain the local extreme values of the function f(x, y) = $x^2 + 2xy$.

j) State Euler theorem.

SECTION-B

2. If $U = \text{Cosec}^{-1}\left(\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right)$, prove that $x^2\frac{\partial^2 U}{\partial y^2} + 2xy\frac{\partial^2 U}{\partial x \partial y} + y^2\frac{\partial^2 U}{\partial y^2} = \frac{13 + \tan^2 U}{144}$.

3. A rectangular box, open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box, requiring least material for its construction.

4. Trace the curve $y^2 = \frac{x-3}{x^2 - 6x + 7}$.

5. a) Find all the asymptotes of the curve $r\sin\theta = a\cos 2\theta$

b) Find centre of gravity of a plate whose density $\rho(x,y)$ is constant and is bounded by the curves $y = x^2$ and $y = x + 2$. Also find moment of inertia about x-axis.

SECTION-C

6. a) Evaluate the integral by changing the order of integration $\int_0^1\int_x^1 \sin y^2 dy dx$.

b) Evaluate the following $\int_0^1\int_x^1 \frac{x\,dx\,dy}{\sqrt{x^2 + y^2}}$.

7. Verify Stoke's Theorem for :

$\vec{F} = (x+y)\hat{i} + (2x+z)\,\hat{j} + (y+z)\,\hat{k}$ for the surface of triangular lamina with vertices (2, 0, 0): (0, 3, 0); (0, 0, 6).

8. Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ax$.

9. a) Prove that $\nabla.\left(\vec{A} \times \vec{B}\right) = \vec{B}.\left(\nabla \times \vec{A}\right) - \vec{A}.\left(\nabla \times \vec{B}\right)$.

   b) Evaluate $\int_c \left(x^2 + xy\right)dx + \left(x^2 + y^2\right)dy$, where c is the square formed by the lines $x = \pm$ l, $y = \pm$l.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.