

Roll No.

Total No. of Pages : 03

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B.Tech. (AE/A&R/CSE/EEE/IT/ME/CE/ME/ECE/EE) (Sem-1)

**MATHEMATICS-I**

Subject Code : BTAM-101-18

M.Code : 75353

Date of Examination : 15-06-2023

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

**SECTION-A**

1. Write briefly :

a) What is maximum value of function  $f(x) = \sin x + \cos x$

b) Evaluate  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(x \cos x)}{\cos(x \sin x)}$ .

c) Find the equation of tangent plane to the surface  $xyz = 6$  at  $(1,2,3)$ .

d) Show that the function  $f(x,y) = \begin{cases} 2x^2 + y; & (x,y) \neq (1,2) \\ 0; & (x,y) = (1,2) \end{cases}$  is discontinuous at  $(1, 2)$ .

e) Calculate approximate value of  $\sqrt{24}$  to two decimal places by Taylor's theorem.

f) Evaluate  $\int_0^1 \int_1^2 (x+5) dy dx$

g) Examine the nature of the series  $1 + 2 + 3 + \dots + n + \dots + \infty$

h) Define skew symmetric matrices with example.

- i) Find the characteristic equation of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$
- j) Find the eigen value of the matrix  $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ .

### SECTION-B

2. a) Verify Rolle's Theorem for  $f(x) = \sqrt{4-x^2}$  in  $[-2, 2]$ .
- b) Apply Taylor's theorem with Lagrange's remainder to function  $f(x) = \sin x$  in  $\left[\frac{\pi}{2}, x\right]$ .
3. Discuss the convergence of the following improper integral
- (a)  $\int_0^{\infty} \frac{1}{(x^2+a^2)(x^2+b^2)} dx, a \neq b$       (b)  $\int_2^3 \frac{x+1}{\sqrt{x-2}} dx$
4. Find the shortest distance between the line  $y = 10 - 2x$  and the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ .
5. a) Evaluate by changing the order of integration of  $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy dx$ .
- b) Find the volume common to the sphere  $x^2 + y^2 + z^2 = a^2$  and the cylinder  $x^2 + y^2 = ay$ .

### SECTION-C

6. a) Discuss the convergence or divergence of the series  $\sum \frac{\sqrt{x+n-1}}{\sqrt{x^2+n^2+1}}$ .
- b) Test the convergence of  $\sum \frac{(1+nx)^n}{n^n}$ .

7. a) Test the convergence of  $x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \dots \rightarrow \infty$

b) Test the convergence of  $\sum \frac{1}{\left(1 + \frac{1}{n}\right)^{n^2}}$ .

8. Find the value of  $\lambda$  for which the equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$ ,  $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$ ,  $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$  are consistent and find the ratios of  $x, y, z$  when  $\lambda$  has the smallest of these values. What happens when  $\lambda$  has the greatest of these values.

9. a) Find a matrix  $B$  which transforms  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$  into a diagonal form.

b) Find the rank of the matrix  $\begin{bmatrix} 1 & -2 & 2 & 3 & 6 \\ 0 & -1 & -3 & 1 & 1 \\ -2 & 4 & -3 & -6 & 11 \end{bmatrix}$ .

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**