Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages: 03**

Total No. of Questions: 09

B.Tech (AI&ML / AI and Data Science / CSE / IOT / Data Science / IT / Robotics & AI / Internet of Things and Cyber Security including Block Chain Technology / Computer Engg.) (Sem– 1)

MATHEMATICS-I

Subject Code: BTAM-104-18

M Code: 75362

Date of Examination : 06-06-2023

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each, carrying EIGHT marks each.**
3. **Attempt any FIVE questions from SECTION B & C, selecting atleast TWO questions from each of these SECTIONS B & C.**

SECTION-A

1. Write briefly :

a) State relation between Beta and Gamma functions.

b) Solve $\int_0^{\infty} \frac{x^8\left(1-x^6\right)dx}{\left(1+x\right)^{24}}$

c) Find the area of the region enclosed by the curve $y = x^2$ and the lines $x = 0$, $y = 0$ and $x = 2$.

d) Evaluate $\lim_{x\to 0} \frac{\log x^2}{\cot x^2}$

e) Calculate approximate value $\sqrt{26}$ to two decimal places by Taylor's theorem.

f) Are the vectors (1,2,1), (2,1,4), (1,8,–3), (4,5,6) linearly dependent? If yes, find relation between them.

g) Show that the transformation $T : R^2 \rightarrow R^3$ *defined by* $T(x,y,z) = (x+y, y+z, z+x)$ is linear.

h) Calculate A^5 for the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.

i) Prove that the product of two orthogonal matrices of the same order is orthogonal.

j) State rank Nullity theorem.

SECTION-B

2. a) The curve $y^2(a+x) = x^2(3a-x)$ is revolved about the axis of x. Find the volume generated by the loop.

   b) Find absolute maximum and minimum value of $f(x) = x-2\sin x$ on $[0, 2\pi]$.

3. a) Find rank of $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

   b) Using properties of determinants, evaluate :

$$\begin{bmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{bmatrix}$$

4. a) Solve by Gauss Elimination method the system of equation

   $x+y+z=3$, $\quad 3x-9y+2z=-4$, $\quad 5x-3y+4z=6$

   b) Find the inverse of matrix

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

5. a) State and prove Rodrigue's Duplication formula.

   b) Use Taylor's theorem to express $f(x) = 4x^3 - 5x^2 + 3x - 9$ in the powers of $(x-3)$.

SECTION-C

6. Let $T : R^3 \to R^2$ defined by $T(x,y,z) = (3x + 2y - 4z, x - 5y + 3z)$

   a) Find the matrix of T corresponding to bases of R^2: $B_1 = \{(1,1,1), (1,1,0\}, (1,0,0)$, $B_2 = \{(1,3), (2,5)\}$.

   b) Verify that the action of T is preserved by its matrix representation i.e. $[T; B_1, B_2][v; B_1] = [T(v); B_2]$ for all $v \in R^3$.

7. a) Express the matrix A as sum of symmetric and skew symmetric matrix where

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

b) Examine whether A is similar to B where $A = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.

8. a) Find Eigen values & Eigen Vectors of $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

b) Diagonalize $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

9. a) Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & -3 & -1 \\ -1 & 4 & 2 \\ -1 & 3 & 0 \end{bmatrix}$

b) Let V be a vector space of 2×2 matrices over R and let $M = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$. Let $T: V \to V$ be the linear map defined by $T(A) = MA \; \forall A \in V$. Find the basis and dimension of

i) Null space of T

ii) Range of T.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.