

Roll No.

Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (ME) (Sem-2)

**MATHEMATICS-II**

Subject Code : BTAM203-18

M.Code : 76256

Date of Examination : 16-06-2023

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

**SECTION-A**

1. Solve :

- a) Find general solution of  $y = xy' - (y')^3$ .
- b) Find Integrating Factor of  $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$ .
- c) Find non-ordinary (singular) points of  $(x^2 + 2x)\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} - y = 0$ .
- d) Check exactness of  $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ .
- e) Solve  $x\frac{dy}{dx} + y = x \log x$ .
- f) Write C-R equations in polar co-ordinates.
- g) Show that the function  $u = e^{-2xy}(x^2 - y^2)$  is harmonic.

h) Find the image of the region bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x = 2$ ,  $y = 1$  under the transformation  $w = z + 1 - 2i$ .

i) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1) dz$  along  $x = t + 1$ ,  $y = 2t^2 - 1$ .

j) State Maximum Modulus Theorem.

### SECTION-B

2. a) Find complete solution of the differential equation  $(x^3y^2 + x)dy + (x^2y^3 - y)dx = 0$ .

b) Solve the differential equation  $\frac{d^2y}{dx^2} + \frac{2}{x} \frac{dy}{dx} = \frac{2 \log x^2}{x^2}$ .

3. a) Solve  $y + px = p^2x^4$ .

b) Using the method of variation of parameters, solve  $y'' - 2y' + y = e^x \log x$ .

4. a) Solve :  $\frac{dy}{dx} + 2y = y^2$ .

b) Solve :  $y = 2px + y^2p^3$ .

5. Solve  $(1+x^2)y'' + xy' - y = 0$  in series about  $x = 0$ .

### SECTION-C

6. a) Show that  $f(z) = z|z|$  is not analytic anywhere.

b) Evaluate  $\int_C \tan z dz$ , where  $C$  is  $|z| = 1$ .

7. a) Prove that if  $f(z) = u + iv$  is an analytic function then  $u$  and  $v$  are harmonic functions.

b) Expand  $f(z) = \frac{1}{z^2 + 4}$  as a Taylor's series about  $z = -i$ .

8. a) Show that  $u = \frac{1}{2} \log(x^2 + y^2)$  is harmonic. Determine its analytic function. Find its conjugate also.

b) Evaluate  $\int_C z e^{\frac{1}{z}} dz$  where  $C$  is  $|z| = 1$ .

9. Find the Laurent's series expansion of  $f(z) = \frac{1}{z(z-1)^2}$  in the region  $0 < |z| < 1$  and  $0 < |z| - 1 < 1$ .

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**