

Roll No.

Total No. of Pages : 02

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MCA (Sem-1)
DISCRETE STRUCTURES AND OPTIMIZATION

Subject Code : PGCA-1917

M.Code : 79035

Date of Examination : 25-05-2023

Time : 3 Hrs.

Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying TEN marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Write briefly :

- a) If $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 2x + 2$, $g(x) = 2x - 3$. Find $f \circ g$, $g \circ f$.
- b) Find generating function for series $-5, 25, -125, \dots$
- c) Define B-Tree.
- d) Consider following relation on set $A = \{1, 2, 3\}$, $S = \text{Empty relation}$, $T = \text{Universal Relation}$. Determine whether or not each of above relation on A is an equivalence relation.
- e) Differentiate between POSET and equivalence relation.
- f) Prove that maximum number of edges in a simple graph having n vertices is $n(n-1)/2$.
- g) How many permutations of the letter ABCDEFGH contains the string ABC?
- h) How many edges are there in a tree having n vertices.
- i) Define kernel of a Homomorphism.
- j) Give an example of a relation which is both symmetric and anti symmetric.

SECTION-B

2. a) Let R be relation on the set of ordered pair of positive integers such that $(a,b), (c,d) \in R$ if and only if $a+d=b+c$. Show that R is an equivalence relation.
b) Draw Hasse diagram for divisibility on D_{30} .
3. a) How many bit strings of length ten contains either three consecutive 0s or four consecutive 1s?
b) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D and F.
4. a) Construct circuits from NOT, AND gates and OR gates to produce these outputs.
i) $x y z + x' y' z'$
ii) $((x'+z) (y+z'))'$
b) Let $(A, +, \cdot)$ be a ring such that $a \cdot a = a$ for all a in A . Show that $a + a = 0$ for all a , where 0 is the additive identity. Also show that operation is commutative.
5. Solve recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$.

SECTION-C

6. Show that $\langle \mathbb{Z}, + \rangle$ is a group.
7. a) State and prove Lagrange's theorem.
b) Prove that equality relation is a congruence relation on any algebra
8. a) State and prove Euler's theorem.
b) Show that every connected graph with n vertices has at least $n-1$ edges.
9. a) Draw all subgraphs of the graph with edges $(d,a), (d,c)$ and (d,b) .
b) Determine whether the graph with the edges $(a,b), (a,e), (b,c), (b,d), (b,e), (c,d), (d,e)$ has a Hamilton circuit.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.