Roll No. [ ]

Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (CSE/ME/ECE/CE/IT/EE/EEE) (Sem.–1)

ENGINEERING MATHEMATICS-I

Subject Code : BTAM-101

M.Code : 54091

Date of Examination : 08-12-2023

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Solve the following:

a) Find the curvature of the curve $r = a\sin\theta + b\cos\theta$ at any θ.

b) Find the total length of the curve $r = a\sin^3(\theta/3)$.

c) If $u(x, y) = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, $0 < x, y < 1$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$.

d) Find the relative maximum and minimum values of the function

$f(x,y) = xy + (9/x) + (3/y)$.

e) If $z = f(ax - by)$, then show that $b\frac{\partial z}{\partial x} - a\frac{\partial z}{\partial y} = 0$.

f) Evaluate $\iint_R e^{x^2}\,dx\,dy$, where the region R is given by $R : 2y \le x \le 2$ and $0 \le y \le 1$.

g) Prove that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

h) Find $div\vec{F}$ and $curl\vec{F}$', where $\vec{F} = grad\{x^3 + y^3 + z^3 - 3xyz\}$.

i) Define irrotational vector. Is $\vec{F} = (y + z)\ \hat{i} + (z + x)\ \hat{j} + \{x + y)\ \hat{k}$ an irrotational vector?

j) State Gauss Divergence theorem.

SECTION-B

2. Sketch the graph of the curve $y = \dfrac{(x-1)(x-3)}{x^2}$.

3. a) Find the volume of the solid generated by revolving the region bounded by the curves $y = 3 - x^2$ and $y = -1$ about the line $y = -1$.

   b) Find the surface area of the solid generated by revolving the circle $x^2 + (y - b)^2 = a^2$, $b \geq a$ about the x-axis.

4. If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then show that $\left(\dfrac{\partial f}{\partial x}\right)^2 + \left(\dfrac{\partial f}{\partial y}\right)^2 = \left(\dfrac{\partial f}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial f}{\partial \theta}\right)^2$

5. Find the extreme values of $f(x, y, z) = 2x + 3y + z$, such that $x^2 + y^2 = 5$ and $x + z = 1$.

SECTION-C

6. Evaluate $\iiint_{\Gamma}\sqrt{1 - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2}}\ dxdydz$, where the region is bounded by the

   Curve $\Gamma : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1.$

Prove that $\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^n} \right) = \frac{2-n}{r^n}\vec{a} + \frac{n(\vec{a} \cdot \vec{r})}{r^{n+2}}\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a}$ is a constant vector.

8. Show that $\nabla(\vec{X}.\vec{Y}) = (\vec{X}.\nabla)\vec{Y} + (\vec{Y}.\nabla)\vec{X} + \vec{X} \times (\nabla \times \vec{Y}) + \vec{Y} \times (\nabla \times \vec{X})$.

9. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2dy$, where C is bounded by $y = x$ and $y = x^2$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.