Roll No. | | | | | | | | | | | | | Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (AE/A&R/CSE/EEE/IT/ME/CE/ME/ECE/EE) (Sem–1)

MATHEMATICS-I

Subject Code : BTAM/101/18

M.Code : 75353

Date of Examination : 17-05-2024

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Write briefly :

(a) Verify Rolle's theorem for $f(x) = (x-2)^3(x-3)^4$ in $(-2, 3)$.

(b) Expand $\log(1 + x)$ by Maclaurin's series.

(c) Show that $\sin x (1 + \cos x)$ has a maximum when $x = \pi / 3$.

(d) Find the equation of the tangent plane and normal to the surface $2x^2 + y^2 = 3 - 2z$ at $(2, 1, -3)$.

(e) Change the order of integration of the integral $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} f(x,y)\,dy\,dx$.

(f) Discuss the convergence of the series $\sum_{n=1}^{\infty}\sqrt{\frac{n}{2(n+1)}}$.

(g) Discuss Cauchy's root test for infinite series with example.

(h) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(i) Define orthogonal matrix with example.

(j) Define similar matrices with example.

SECTION-B

2. (a) Prove that $\log (1 + x) = \frac{x}{1+\theta x}$, where $0 < \theta < 1$ and hence deduce that $\frac{x}{1+x} < \log(1+x) < x, x > a.$

(b) Evaluate $\lim_{n\to\infty} \frac{xe^x - \log(1+x)}{x^2}$.

3. (a) Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{3}$.

(b) Find the volume formed by the revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$, about the x-axis.

4. (a) A rectangular box open at the top is to have volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction.

(b) Find the relative maximum and minimum values of the function $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.

5. (a) Find the area bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x = 0$, $y = 0$ in the first quadrant.

(b) Find the mass of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, the variable density $\rho = \mu xyz$.

SECTION-C

6. Discuss the convergence of the series $\frac{x^2}{2\log 2} + \frac{x^3}{3\log 3} + \frac{x^4}{4\log 4} + \ldots\ldots$.

7. (a) Discuss the convergence of $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$.

(b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}x^n$.

8. (a) Investigate the values of λ and μ = so that the equations

$$2x+3y+5z=9,\ 7x+3y-2z=8,\ 2x+3y+\lambda z=\mu$$

have no solution, a unique solution and an infinite number of solutions.

(b) Find the rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 0\\ 2 & 4 & 3 & 2\\ 3 & 2 & 1 & 3\\ 6 & 8 & 7 & 5\end{bmatrix}$.

9. (a) If A is a square matrix show that $A + A^T$ is symmetric and $A - A^T$ skew-symmetric matrix.

(b) Verify the Cayley-Hamilton theorem for the matrix

$A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & -1\\ -2 & -1 & 1\end{bmatrix}$. Find A^{-1} if it exists.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.