Roll No. | | | | | | | | | | | | | **Total No. of Pages : 04**

Total No. of Questions : 09

B.Tech. (AI & ML/DS)(CE)(CSE)(IT)(Robotics & Artificial Intelligence) (Internet of Things and Cyber Security including Block Chain Technology) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM/104/18

M.Code : 75362

Date of Examination : 08-05-2024

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B & C.**

SECTION-A

1. Explain briefly:

a) Verify Lagrange's Mean value theorem for $f(x) = \sin x$ in $[0, \pi]$.

b) Show that $\sin^p \cos^p\theta$ attains a maximum when $\theta = \tan^{-1}(p/q)$.

c) Evaluate $\int_0^\infty e^{-x^2}\,dx$.

d) Determine the rank of the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

e) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, verify that $AA^T = I = A^T A$, where I is the unit matrix.

f) For what values of k, do the following set of vectors forir a basis in $\mathbb{R}^3$:

$\{(k, 1-k, k), (0, 3k-1, 2), (-k, 1, 0)\}$.

g) State $ran(T)$ and $ker(T)$ of a linear transformation $T : V \to W$. State rank-nullity theorem.

h) Examine whether the matrix A is similar to the matrix B where

$$A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}.$$

i) For any square matrix A, show that $A^T A$ is symmetric.

j) Show that the eigen values of an orthogonal matrix are of unit magnitude.

SECTION-B

2. a) Expand $e^{\sin x}$ by Maclaurin's series.

b) Evaluate $\lim_{x\to 0}\frac{(1+x)^{1/x}-e}{x}$.

3. a) Find the volume of the solid formed by revolving about x-axis, the area enclosed by the parabola $y^2 = 4ax$, its involute $27ay^2 = 4(\mathbf{x} - 2a)^3$ and the x – axis.

b) Prove that $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta = \frac{\pi}{\sqrt{2}}$.

4. a) Solve the equations $3x + y + 2z = 3$, $2x - 3y - z = 3$, $x + 2y + z = 4$ by Cramer's rule.

b) Using the Gauss-Jordan method, find the inverse of the matrix

$$A=\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}.$$

5. a) Are the following vectors linearly dependent? If so, find the relation between them : {(1,1,1,3), (1,2,3,4), (2,3,4,9)}.

b) Investigate for what values of λ and μ, the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have no solution, a unique solution and an infinite number of solutions.

SECTION-C

6. a) Let T be a transformation from $\mathbb{R}^3$ into $\mathbb{R}^1$ defined by

$$T(x_1, x_2, x_3) = x_1^2 + x_2^3 + x_3^2.$$

Show that T is not a linear transformation.

b) Find *ker* (T) and *ran* (T) and their dimensions for

$$T: \mathbb{R}^2 \to \mathbb{R}^3; T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x + y \\ y - x \\ 3x + 4y \end{pmatrix}.$$

7. a) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a liner transformation defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y + z \\ y - z \end{pmatrix}$.

Determine the matrix of the linear transformation T, with respect to the standard basis $X = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ in $\mathbb{R}^3$ and

$$Y = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \text{ in } \mathbb{R}^2.$$

b) If x, y, z are linearly independent vectors in $\mathbb{R}^3$ then show that $x + y, y + z, z + x$ are also linearly dependent in $\mathbb{R}^3$.

8. a) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

b) The eigen vectors of a 3 × 3 matrix A corresponding to the eigen values 1, 1, 3 are $(1, 0, -1)^T$, $(0, 1, -1)^T$, $(1, 1, 0)^T$, respectively. Find the matrix A.

9. Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence find P such that $P^{-1} AP$ is a diagonal matrix. Thus, obtain the matrix $B = A^2 + 5A + 3I$.

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