

Roll No.

Total No. of Pages :03

Total No. of Questions : 09

**B.Tech. (AI&ML/CSE/ AI & Data Science/AI/CSE/IOT/IT/Data  
Science/Internet of Things and Cyber Security including Block Chain  
Technology/) (Sem.-4)**

**DISCRETE MATHEMATICS**

**Subject Code : BTCS/401/18**

**M.Code : 77626**

**Date of Examination : 09-05-2024**

**Time : 3 Hrs.**

**Max. Marks : 60**

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

**SECTION-A**

**1. Answer briefly :**

- a) Give an example of a relation which is reflexive, transitive, but not symmetric.
- b) How many people among 200000 people are born at same time (hour, minute, seconds).
- c) How many 8 – letter words can be made using the letters of the words "TRIANGLE", if each word is to begin with T and end with E?
- d) State and Prove Idempotent Laws of Logic Of Proposition.
- e) Define integral domain (I.D).
- f) Find k, if a regular graph with 7 vertices has 12 edges.
- g) Define minimal spanning tree.

- h) Draw a multigraph G whose adjacency matrix  $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

- i) Define cut vertex with an example.
- j) Define POSET.

### SECTION-B

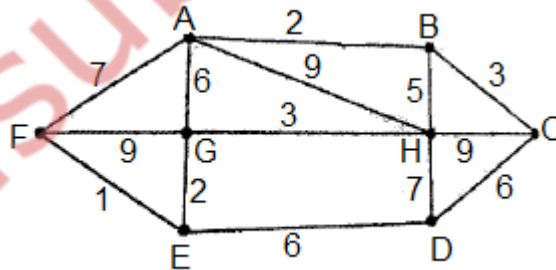
2. Show that  $R$  is a relation in the set  $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ , given by  $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ . Prove that  $R$  is an equivalence relation. Find the set of all elements related to 1 in each case.
3. a) The number of diagonals of a polygon is 20. Find the number of its sides.  
b) Find the number of positive integers from 1 to 1000 which are divisible by none of 5, 6 and 8.
4. a) Prove that  $(p \wedge q) \rightarrow (p \vee q)$  is a tautology.  
b) Test the validity of the following argument:

If my brother stands first in the class, I will give him a watch. Either he stood first or I was out of station. I did not give my brother a watch this time. Therefore I was out of station.

5. Show that if the quotient group  $G/H$  is abelian, then  $G$  may not be abelian.
6. Prove that the number of edges in a complete graph with  $n$  vertices is  $\frac{n(n-1)}{2}$ .

### SECTION-C

7. Use Kruskal algorithm to find spanning tree of minimal weight by showing each step.



8. A subgroup  $H$  of a group  $G$  is a normal subgroup of  $G$  iff the product of two right cosets of  $H$  in  $G$  is again a right coset of  $H$  in  $G$ .

9. a) Prove that the function  $f: \mathbb{C} \rightarrow \mathbb{R}$ , defined by  $f(z) = |z|$  is neither one - one nor onto.
- b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be a real valued function defined by  $f(x) = 2x^3 - 1$ ,  $x \in \mathbb{R}$  and  $g(x) = \left[\frac{x+1}{2}\right]^{\frac{1}{2}}$ ,  $x \in \mathbb{R}$ . Show that  $f$  and  $g$  are bijective.

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**