

Roll No.

Total No. of Pages : 03

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B.Tech. (AE/A&R/CSE/EEE/IT/ME/CE/ME/ECE/EE)(Sem-1)

**MATHEMATICS-I**

Subject Code : BTAM/101/18

M.Code : 75353

Date of Examination:24-12-2024

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

**SECTION-A**

1. Solve :

a) Verify Rolle's Theorem for  $f(x) = x^2 - 4x + 3$  in the interval  $[1, 3]$ .

b) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$ .

c) Evaluate  $\int_1^{\infty} \frac{1}{x^2} dx$ .

d) Find  $\frac{\partial f}{\partial x}$  for  $f(x, y) = x^3 + 3xy + y^2$ .

e) Evaluate  $\int_0^{\pi/4} \int_0^{\cos \theta} r dr d\theta$  by converting to Cartesian coordinates.

f) Analyze if the series  $\sum_{n=1}^{\infty} \frac{1}{n^3 + 1}$  converges.

g) Check the convergence of the series  $\sum_{n=1}^{\infty} \frac{n}{2^n}$ .

h) Write the Taylor series for  $f(x) = \sin(x)$  centered at  $x = 0$  up to the second term.

i) Examine whether the matrix  $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  is orthogonal.

j) Determine whether the matrix  $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$  has an inverse or not.

### SECTION-B

2. a) Find the critical points and determine the maximum and minimum values of  $f(x) = x^3 - 6x^2 + 9x + 5$ .

b) Expand  $\ln(1+x)$  using the Maclaurin series up to the third-degree term.

3. Calculate the volume of the solid generated by revolving the region bounded by  $y = \sqrt{x}$  and  $y = 0$  from  $x = 0$  to  $x = 4$  about the  $x$ -axis.

4. Find the maximum and minimum values of  $f(x, y) = x^2 + y^2$  subject to the constraint  $x + y = 1$ .

5. Find the center of mass of a semicircular plate of radius  $r$  with variable density  $\rho(x, y) = y$ .

### SECTION-C

6. Discuss the convergence of  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$  using the alternating series test, explaining the conditions necessary for this test.

7. Find the radius and interval of convergence for the series  $\sum_{n=10}^{\infty} \frac{x^n}{2n\sqrt{n+1}}$ .

8. Use the Cayley-Hamilton theorem to find  $A^{-1}$  for the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$ , if its invertible.

9. Verify rank-nullity theorem for the matrix  $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{pmatrix}$ .

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**