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Total No. of Questions : 09

B.Tech. (AI & ML) (AI & DS) (DS) (CE)(CSE)(IT) (Internet of Things and Cyber Security including Block Chain Technology) (Sem.–1)

MATHEMATICS-I (CALCULUS & LINEAR ALGEBRA)

Subject Code : BTAM-104-18

M.Code : 75362

Date of Examination:02-01-2026

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B & C.**

SECTION-A

1. Explain briefly:

a) Find the surface area of the solid obtained by rotating the curve $y = \sqrt{x}$ from $x = 1$ to $x = 4$ about the x-axis.

b) Apply Taylor's theorem to approximate $f(x) = \ln(x + 1)$ up to the second-degree term around $x = 0$.

c) Evaluate the limit: $\lim\limits_{x \to 0} \dfrac{x - \sin(x)}{x^3}$.

d) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

e) Find the value of $3A + 2B$, where $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$.

f) Calculate the nullity of the linear transformation whose associated matrix is $A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 3 \\ 4 & -1 & 0 \end{pmatrix}$.

g) Examine if {(1, 2, 3), (4, 5, 6), (7, 8, 9)} is linearly dependent in $\mathbb{R}^3$.

h) Find the eigenvector corresponding to the eigenvalue $\lambda = 3$ for the matrix $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$.

i) Given the matrix $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$, check if it is an orthogonal matrix.

j) Prove that the diagonal elements of a real skew-symmetric matrix are always 0.

SECTION-B

2. a) Find the critical points and determine the maximum and minimum values of $f(x) = (x^2 - 1)^3$.

   b) Prove that $\Gamma(1/2) = \sqrt{\pi}$.

3. Evaluate the integral $\int_0^1 x^{1/2} (1 - x)^{1/3}\, dx$.

4. Verify Taylor's theorem with remainder for $f(x) = \sin(x)$ at x = 0 up to the fourth degree, and approximate sin(0.5).

5. **Use Gauss elimination to solve the system of linear equations:**

$$x + y + z = 6,\ 2x - y + 3z = 14, -x + 2y + 2z = -2.$$

SECTION-C

6. For the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x + 2y - z, 3x - y + z)$, determine the range space and null space of T and verify the rank-nullity theorem.

7. Let $T:\ \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map defined by T(x, y) = (3x + 4y, 5x + 6y). Find the matrix associated with T.

8. Let $A = \frac{1}{7}\begin{pmatrix} 3 & 2 & 6 \\ -6 & 3 & 2 \\ 2 & 6 & -3 \end{pmatrix}$.

   a) Show that A is an orthogonal matrix.

   b) Find the inverse of A using the property of orthogonal matrices.

9. Diagonalize the matrix $\begin{pmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{pmatrix}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.